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Usage Spectrum Perturbation Effects on Helicopter Component Fatigue Damage and Life-Cycle Costs

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ABSTRACT

The Australian Defence Force (ADF) operates rotary and fixed-wing aircraft in a different manner from the way that the aircraft manufacturers envisaged. That is, the usage spectra of the ADF aircraft are different from the usage spectra assumed by the aircraft manufacturer during design. The effects of perturbing the amount of time spent in flight conditions on both component fatigue damage and life-cycle costs are investigated in this report. An "amplification factor" is developed, which allows the effect of varying the amount of time spent in different flight conditions on both damage and cost to be determined. These amplification factors give both qualitative ("importance" ordering) and quantitative (sensitivity) information regarding all flight conditions. The resulting procedure is both easy to implement and use, and allows the operator to determine the effects of different spectra on both damage and cost. The outlined procedures will lead to cost savings and safety improvement for the ADF for both rotary and fixed-wing aircraft.

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EXECUTIVE SUMMARY

When designing an aircraft, be it fixed or rotary-wing, the design engineer assumes a flight usage spectrum for the aircraft. In practice, the actual usage spectrum will differ from the design usage spectrum.

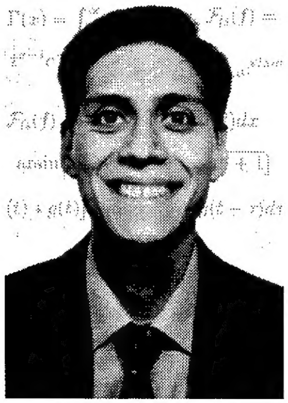
At present it is difficult to quantify the effects, if any, that changes to the original design spectrum have on component fatigue damage rates, and hence on component lives and life-cycle (LC) costs. This report investigates usage spectrum perturbation effects. In particular, the effect of changing the fraction of time spent in flight conditions, on damage and LC costs, is investigated. Only the component purchase costs are considered in this analysis; other costs such as those applicable to the labour required to remove and install components, for example, were not taken into consideration. Although only damage and LC costs were investigated, the techniques developed can just as easily be applied to other types of analyses such as reliability.

Using what we term "amplification factors", we can prioritise flight conditions according to "importance". The method we develop, however, is not merely limited to a qualitative analysis, since these amplification factors also give us quantitative information in the form of sensitivity of damage or LC costs to usage spectrum perturbations. Using a simple Black Hawk helicopter example, we investigate the effects on damage and LC costs for five different components resulting from the variation of fractional time spent in 17 different flight conditions.

We show that the changes in damage and LC costs resulting from a change in the proportion of time spent in a given flight condition is inversely proportional to that change in the time in the flight condition, and proportional to the fractional damage or fractional LC costs respectively. We show that, for small changes in flight condition times, the fatigue damage and LC costs are mildly affected for a few flight conditions, and almost unaffected for most flight conditions. However, the time spent in a few flight conditions may be subject to large variations from that assumed in the design usage spectrum, and the resulting fatigue damage may be of concern to operators.

This report outlines a simple method of determining the effects of usage spectrum perturbations on component fatigue damage and LC costs for the ADF.

Author



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Frank Polanco graduated in 1992 with a Bachelor of Aerospace Engineering (Honours) and a Bachelor of Applied Science (Distinction) from the Royal Melbourne Institute of Technology (RMIT). He joined the Aeronautical and Maritime Research Laboratory (AMRL) in 1993, working on aircraft structural integrity and fatigue life monitoring before returning to RMIT to complete a Doctorate in Mathematics. He then rejoined the AMRL in 1998 to work in the area of helicopter life assessment.

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1 Introduction

The usage spectrum flown by operators is rarely the same as that initially envisaged for the aircraft by the manufacturer. This difference between actual and design usage spectra may lead to premature fatigue failure or premature retirement of some components [1, 2, 5, 6], resulting in both safety and cost concerns. It would be beneficial to determine the impact (on component fatigue damage, life-cycle costs, safety, etc.) of any perturbation from the design usage spectrum. Although in this report only life-cycle (LC) costs and component fatigue damage analyses are considered, these same principles may be used, for example, in reliability (or safety) analysis.

We develop an *amplification factor* that allows flight conditions to be easily ordered in terms of their effect on fatigue damage and LC costs. Note that the LC costs considered here are based solely on the purchase price of the component. No attempt has been made to include other costs, such as that for the labour required to change the component.

King and Lombardo [4] investigate, amongst other things, two topics (related to Black Hawk helicopters) that are closely related with this report; (i) the sensitivity of fatigue damage rates to variations in helicopter usage and (ii) the relationship between fatigue spectrum and the cost of ownership. King and Lombardo provide data for "the rapid estimation" of these sensitivities and costs. The estimation procedure they use involves *re-calculating* the fatigue damage and cost for each new usage spectrum configuration. In this report we assume that the original fatigue damage and cost analyses are correct, and hence only investigate *percentage changes* of these quantities.

The differences between the two reports are now summarised. In King and Lombardo's report [4]:

- The S-70A-9 Black Hawk helicopter, in particular, is investigated and hence the report is strong on detail.
- Gaining an overall appreciation of different usage spectrum modifications would be difficult because of the "what-if" approach. The "what-if" approach necessarily leads to a combinatorial explosion in the number of possible usage spectrum modifications.

In contrast, this report:

- Develops a mathematical model of usage spectrum modifications, and is hence more generic. (For the purposes of illustration only a simplistic usage spectrum is used.)
- The development of "amplification factor" tables allows the easy comparison of different usage spectrum modifications (including complex combinations of modifications).
- The mathematical development also provides insight into why fatigue damage and costs are particularly sensitive to modifications in some flight conditions and not others.

We begin the core of this report by defining the design and actual usage spectra in § 2, and provide a concrete example in the form of a simplified usage spectrum for the Black Hawk helicopter. In § 3 we investigate the effects of spectrum modifications on

LC costs. We also determine, for a given range of modifications, what the maximum and minimum costs are and where they occur. An example of modifying three flight conditions is given in § 4, and for these same three flight conditions we also determine the magnitude and location of the cost extrema. Special cases of the above analysis are considered in § 5, namely the effect of modifying the proportion of time spent in any number of flight conditions by the same amount and of modifying the proportion of time spent in only one flight condition.

In § 6 we briefly derive the result of spectrum modification on damage, using the results from the analysis on cost. The ordering of cost analysis before damage analysis sounds back-to-front. However, given a valid table of damage rates and costs for a defined usage spectrum, we will see that cost and damage analyses are (in some sense) independent. This independence follows from the fact that for the cost analyses, damage has already been taken into account in the given damage table. The reason the cost analysis is presented first is that mathematically it is a little simpler. In that section, § 6, we also tabulate amplification factors (which quantify the effects of changing the amount of time spent in flight conditions) for two examples.

The final section of this report, § 7, summarises the results and draws conclusions from our findings.

2 Design and Perturbed Usage Spectra

We start by defining two usage spectra, a design (or unmodified) usage spectrum and a perturbed (or modified or new) spectrum. As a concrete example, consider the usage spectrum shown in Table 2.1, which shows the fatigue damage experienced by components in various flight conditions [4]. This spectrum is actually a condensed version of the U.S. Army UH-60A Black Hawk helicopter design usage spectrum.

Table 2.1: Design usage spectrum including time fraction, component fatigue damage, and life-cycle costs. Component name, number, and purchase cost are shown above the main table. The following abbreviations have been used in this table: partial power descent (ppd), autorotation (autos), sideward and rearward flight (side/rear), control reversal (reversal), droop stop pounding (dsp), ground-air-ground cycles (gag cycles), and heavy manoeuvres (heavy man.); see King and Lombardo [4] for further details.

		Compnt	1.Main rotor hub	2.Main support bridge	3. Main rotor blade	4.Lateral bell crank	5.Expand-able blade pin	
		Cost	$190,141	$30,376	$161,871	$2,691	$900	
No.	**Flight condition**	**Time (hr)**	**Damage / 100 hr**					**Cost / 100 hr**
1	taxi	0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 0.00
2	take off	1.765	4.79E-6	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 0.91
3	hover	2.32	1.03E-5	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 1.95
4	climb	4.198	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 0.00
5	ppd	2.5	8.88E-6	4.75E-5	0.00E+0	1.15E-5	3.11E-5	$ 3.19
6	dive	2.324	0.00E+0	1.79E-5	0.00E+0	2.29E-5	0.00E+0	$ 0.61
7	autos	1.781	0.00E+0	2.99E-6	0.00E+0	0.00E+0	9.77E-6	$ 0.10
8	level flt	71.36	0.00E+0	5.34E-5	0.00E+0	4.63E-5	0.00E+0	$ 1.75
9	turns	9.91	1.13E-5	3.80E-4	0.00E+0	7.45E-5	2.40E-5	$13.92
10	side/rear	1.5	1.60E-5	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 3.05
11	sideslip	1	1.22E-5	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 2.32
12	pullouts	0.306	8.65E-6	3.33E-5	1.97E-6	1.78E-5	2.38E-5	$ 3.04
13	reversals	0.504	1.48E-5	1.66E-5	0.00E+0	0.00E+0	8.20E-7	$ 3.32
14	landing	0.362	4.11E-6	0.00E+0	0.00E+0	0.00E+0	0.00E+0	$ 0.78
15	dsp	0.139	4.47E-5	0.00E+0	0.00E+0	0.00E+0	1.93E-5	$ 8.52
16	gag cycle	0.03086	3.47E-5	1.21E-5	1.36E-4	4.94E-6	6.14E-5	$29.03
17	heavy man.	0.00014	3.90E-7	0.00E+0	1.09E-6	0.00E+0	0.00E+0	$ 0.25
Total		100	1.71E-4	5.64E-4	1.39E-4	1.78E-4	1.70E-4	$72.74

The second column contains the flight condition type, the third column the time spent in that flight condition (per 100 flight hours), and the last column shows the incremental amount each flight condition contributes to the LC costs for the five components (per 100 flight hours). Columns four to eight list the fatigue damage experienced by a particular component per 100 flight hours for a given flight condition. The small sub-table (above the fatigue damage table) lists the purchase costs for the five components. This information is used to calculate the overall cost attributable to each flight condition by multiplying each fatigue damage value by the appropriate component cost. This follows since fatigue damage is defined as the fraction of life expended; that is, the component is replaced when the total accumulated fatigue damage value reaches unity.

We now introduce some notation to generalise the process of usage spectrum perturbations. Let D_{ij} denote the *fatigue damage* that is accumulated over a given period of time, T, where the subscripts i and j represent the component and flight condition respectively. For example, $D_{2,5}$ is the fatigue damage produced by the fifth flight condition on the second component (in the case of Table 2.1, $D_{2,5} = 4.75{\times}10^{-5}$). Also, let t_j and $\$_j$ be the *time* and *LC costs* of the jth flight condition respectively (again from Table 2.1, $t_5 = 2.5$ and $\$_5 = 3.19$). As we see from Table 2.1, the simple relation between flight condition LC costs and flight condition fatigue damage is

$$\$_j = \sum_{i=1}^{I} D_{ij} C_i,$$

where I is the number of components (five in Table 2.1) and C_i is the cost of the ith component. Further, the sum of all flight condition times is T, and the sum of all flight condition LC costs is \$, that is

$$\sum_{j=1}^{J} t_j = T \tag{2.1}$$

and

$$\sum_{j=1}^{J} \$_j = \$, \tag{2.2}$$

where J is the number of flight conditions in the usage spectrum (in Table 2.1, $T = 100$, $\$ = 72.74$, and $J = 17$).

Note that the some flight conditions (gag cycles for example) are normally defined in terms of occurrences. In the table above these occurrences were converted to fraction of flight time by multiplying the number of occurrences by the amount of time spent in those occurrences (per 100 flight hours). Mathematically, let the jth flight condition occur $\bar{n}_j$ times (per T hours), and a single occurrence of this flight condition take $\bar{t}_j$ hours with corresponding damage $\bar{D}_{ij}$ of the ith component. The amount of fractional time and component damage that this flight condition consumes (per T hours) are $t_j = (\bar{n}_j\, \bar{t}_j)/T$ and $D_{ij} = \bar{n}_j\, \bar{D}_{ij}$ respectively. We now see that the fractional amount of time spent in a single occurrence of these flight conditions may be arbitrarily set to some small number (that is $\bar{t}_j/T \ll 1$). The small magnitude of fractional time means that the remaining flight

conditions are unaffected by changes to the occurrence counted flight condition. However, changes to the number of occurrences still have an effect in terms of damage.

All variables denoting the perturbed spectrum will be hatted, so that for example $\widehat{t}_5$ will be the time spent in the perturbed spectrum's fifth flight condition. Introducing ϵ_j as the amount by which the time spent in the jth flight condition increases, yields the following relation between new and old times

$$\widehat{t}_j = t_j(1+\epsilon_j).$$

For example, if the time in the fifth flight condition is decreased by 17% then $\epsilon_5 = -0.17$, while if the time in the ninth flight condition is increased by 150% then $\epsilon_9 = 1.50$. In addition, bound all variation of a given flight condition by $\epsilon_j \in [\epsilon_j^-, \epsilon_j^+]$, where $\epsilon_j^- \leq 0$ and $\epsilon_j^+ \geq 0$ are the jth flight condition's lower and upper bounds respectively. All modified times must still be non-negative $(\widehat{t}_j \geq 0)$, so the lower bounds must also satisfy the relation

$$\epsilon_j^- \geq -1 \qquad \text{for all } j = 1, 2, \ldots, J. \tag{2.3}$$

Also, the sum of all modified new times must not be greater than the total time under consideration

$$\sum_{j=1}^{k} t_j(1+\epsilon_j^+) \leq T, \tag{2.4}$$

where k is the number of flight conditions we are modifying. (For example, if we only modified landings, turns, and dives, then $k = 3$.) There may be additional physical considerations restricting the upper and lower bounds (ϵ_j^- and ϵ_j^+) even further; for example the number of landings and take-offs must be the same. The restriction given by Equation (2.4) on the upper bounds of time variations, ϵ_j^+, was chosen to simplify the analysis for the maximum and minimum points, and, as such, may be relaxed (we postpone a more detailed discussion of this relaxation until § 3). Finally, we also require that the times of the modified spectrum still sum to the same total time under consideration, and thus

$$\sum_{j=1}^{J} \widehat{t}_j = T. \tag{2.5}$$

Now, assume we wish to modify the time of the first k flight conditions, where $k < J$ (we will solve for the special case $k = J$ at a later stage). Note that we have not lost any generality in choosing the first k terms instead of any k terms, since we may order the J usage spectrum's flight conditions in any way we desire. Let the remaining $J - k$ flight conditions be adjusted by the same proportional constant ϵ (which we term the *average variation* of the design flight conditions), that is

$$\epsilon_j = \epsilon \qquad \text{for } j = k+1, k+2, \ldots, J,$$

so that Equation (2.5) is satisfied. Then we have that

$$\widehat{t}_j = \begin{cases} t_j(1+\epsilon_j), & j = 1, 2, \ldots, k, \\ t_j(1+\epsilon), & j = k+1, k+2, \ldots, J. \end{cases}$$

Thus we may solve for ϵ starting from Equation (2.5)

$$\begin{aligned} T &= \sum_{j=1}^{J} \widehat{t}_j \\ &= \sum_{j=1}^{J} t_j(1+\epsilon_j) \\ &= \sum_{j=1}^{J} t_j + \sum_{j=1}^{J} t_j\epsilon_j \\ T - \sum_{j=1}^{J} t_j &= \sum_{j=1}^{k} t_j\epsilon_j + \epsilon \sum_{j=k+1}^{J} t_j. \end{aligned} \tag{2.6}$$

But from Equation (2.1) we know that $T - \sum_{j=1}^{J} t_j = 0$ and hence $\sum_{j=k+1}^{J} t_j = T - \sum_{j=1}^{k} t_j$, using these two relations Equation (2.6) becomes

$$0 = \sum_{j=1}^{k} t_j\epsilon_j + \epsilon \left(T - \sum_{j=1}^{k} t_j \right).$$

Dividing through by T and solving for the average variation ϵ we obtain

$$\epsilon = -\frac{\displaystyle\sum_{j=1}^{k} \tau_j\epsilon_j}{1 - \displaystyle\sum_{j=1}^{k} \tau_j}, \tag{2.7}$$

where τ_j is the *fractional time* spent in the jth flight condition defined by

$$\tau_j = \frac{t_j}{T}.$$

Multiplying the fractional time by 100 gives percentage time spent in a flight condition. We now see why we couldn't modify all flight conditions at once (that is, we could not set $k = J$), since doing so would yield a zero denominator in the above expression. If we alter the fractional times of all flight conditions, then there are no flight conditions left over to automatically modify by the average variation ϵ. In other words, since we have to satisfy the restriction $\sum_{j=1}^{k} t_j(1+\epsilon_j) = T$, the variation of at least one flight condition is pre-determined if we vary the remaining $J-1$ flight conditions. This non-zero condition, for the denominator, assures a finite value of the average variation ϵ in Equation (2.7).

Note that the equation shown above for ϵ only requires knowledge of the fractional time of the flight conditions we are modifying, not all flight conditions. So that, for example, if we only modify two flight conditions, the evaluation of ϵ would only require two terms in the summations of Equation (2.7). This represents a considerable simplification, since only the terms that are modified are used in the calculations. For completeness, we also require that at least one of the design spectrum's flight conditions has a non-zero time in

the initial spectrum, that is $\sum_{j=k+1}^{J} \tau_j \neq 0$. With the above analysis we have now satisfied the time constraint $\sum_{j=1}^{J} \widehat{t}_j = T$.

We will show below that the modified spectrum's damage, and hence cost, may be written in terms of the fractional time variations (ϵ_j). Miner's Rule (King *et al.* [3, pp. 28–29]) expresses fatigue damage as

$$D = \frac{n}{N},$$

where n is the number of load cycles experienced by the component so far, and N is the allowed number of cycles, after which the component should be retired (N is calculated from the working S-N curve). The number of load cycles experienced is given by

$$n = c\tau_j,$$

where c is a constant and is the average frequency of the application of the loads to the particular component during the given flight condition. Thus the increase in fatigue damage is directly proportional to each flight condition's fractional time increase, that is,

$$\widehat{D}_j = D_j(1 + \epsilon_j),$$

and similarly for the LC costs of each flight condition under the modified spectrum

$$\widehat{\$}_j = \$_j(1 + \epsilon_j).$$

Finally define the total LC costs of the new usage spectrum as the *new total LC costs*

$$\widehat{\$} = \sum_{j=1}^{J} \widehat{\$}_j. \tag{2.8}$$

3 Modified Spectrum's Fractional Increase in Life-Cycle Costs

In this section we investigate the effects of spectrum modifications on component and total costs. Several assumptions lie beneath the single dollar value calculated for the total cost. We now outline some deficiencies in this single dollar value analysis. In reality costs are incurred when components are replaced, and so where component lives approach service lives the costs are likely to vary in discrete jumps. Also, a rigorous evaluation of costs would need to take the time factor into account (paying for something today is not the same as having to pay for it some time in the future). The dollar cost value developed in this section approaches the true cost only when the rate-of-effort is high and components are replaced many times during the service life of the aircraft. As such, dollar values developed using the procedure described in this section are only indicative of which flight condition perturbations lead to large cost changes.

We want to investigate what effect modifications to the usage spectrum have on LC costs. One measure of this effect is given by the *fractional increase in LC costs*, which is given mathematically by $(\widehat{\$/\$}) - 1$. We begin by deriving the LC costs of the new usage spectrum in terms of the LC costs of the design usage spectrum.

Partitioning the new LC costs, given by Equation (2.8), at the kth term gives

$$\widehat{\$} = \sum_{j=1}^{k} \$_j(1+\epsilon_j) + \sum_{j=k+1}^{J} \$_j(1+\epsilon).$$

Dividing the above expression by the LC costs, \$, introducing the *fractional LC costs* of jth flight condition as

$$\sigma_j = \frac{\$_j}{\$},$$

and using the relation $\sum_{j=k+1}^{J} \sigma_j = 1 - \sum_{j=1}^{k} \sigma_j$, which is derived from Equation (2.2), gives

$$\begin{aligned}
\frac{\widehat{\$}}{\$} &= \sum_{j=1}^{k} \sigma_j(1+\epsilon_j) + (1+\epsilon)\left(1 - \sum_{j=1}^{k} \sigma_j\right) \\
&= \sum_{j=1}^{k} \sigma_j + \sum_{j=1}^{k} \sigma_j\epsilon_j + 1 - \sum_{j=1}^{k} \sigma_j + \epsilon\left(1 - \sum_{j=1}^{k} \sigma_j\right),
\end{aligned}$$

and hence

$$\frac{\widehat{\$}}{\$} - 1 = \sum_{j=1}^{k} \sigma_j\epsilon_j + \epsilon\left(1 - \sum_{j=1}^{k} \sigma_j\right). \tag{3.1}$$

Define the *LC costs fraction* and *new LC costs fraction* respectively as

$$\sigma = \sum_{j=1}^{k} \sigma_j \tag{3.2}$$

and

$$\widehat{\sigma} = \sum_{j=1}^{k} \sigma_j (1 + \epsilon_j). \tag{3.3}$$

(Note that we are only summing over the modified flight conditions.) Similarly define the *total time fraction* and *new total time fraction* respectively as

$$\tau = \sum_{j=1}^{k} \tau_j \tag{3.4}$$

and

$$\widehat{\tau} = \sum_{j=1}^{k} \tau_j (1 + \epsilon_j). \tag{3.5}$$

Using these definitions of the time fractions the average variation, given by Equation (2.7), becomes $\epsilon = -(\widehat{\tau} - \tau)/(1 - \tau)$, and hence the effect on LC costs, given by Equation (3.1), is

$$\begin{aligned} \frac{\widehat{\$}}{\$} - 1 &= (\widehat{\sigma} - \sigma) - \left(\frac{1-\sigma}{1-\tau}\right)(\widehat{\tau} - \tau) \\ &= (\widehat{\sigma} - \sigma) - \alpha_k(\widehat{\tau} - \tau) \end{aligned} \tag{3.6}$$

where

$$\begin{aligned} \alpha_k &= \frac{1-\sigma}{1-\tau} \qquad (3.7) \\ &= \frac{1 - \sum_{j=1}^{k} \sigma_j}{1 - \sum_{j=1}^{k} \tau_j} = \frac{\sum_{j=k+1}^{J} \sigma_j}{\sum_{j=k+1}^{J} \tau_j} \end{aligned}$$

is the *ratio of design fractions*, which is constant (for any given set of flight conditions to be modified) since σ and τ only refer to the design spectrum. (The ratio of design fractions is simply the ratio of design fractional LC costs to design fractional times.) Defining the *amplification factors* as

$$\begin{aligned} \gamma_j &= \frac{\$_j}{\$} - \alpha_k \frac{t_j}{T} \\ &= \sigma_j - \alpha_k \tau_j \end{aligned} \tag{3.8}$$

for $j = 1, 2, \ldots, k$, we can re-write the fractional increase in LC costs (3.6) as

$$\begin{aligned} \frac{\widehat{\$}}{\$} - 1 &= \left[\sum_{j=1}^{k} \sigma_j(1+\epsilon_j) - \sum_{j=1}^{k} \sigma_j\right] - \alpha_k \left[\sum_{j=1}^{k} \tau_j(1+\epsilon_j) - \sum_{j=1}^{k} \tau_j\right] \\ &= \sum_{j=1}^{k} (\sigma_j - \alpha_k \tau_j)\, \epsilon_j \\ &= \sum_{j=1}^{k} \gamma_j \epsilon_j. \end{aligned} \tag{3.9}$$

We see from the above relation that LC costs vary linearly with each modified flight condition's fractional time increase ϵ_j. Also note that the amplification factors γ_j play an important role in the fractional LC costs increase. For an increase in the fractional time of the jth flight condition ($\epsilon_j > 0$), it is the sign of the jth amplification factor γ_j that determines whether the fractional LC costs increases ($\gamma_j > 0$), decreases ($\gamma_j < 0$), or remains the same ($\gamma_j = 0$). Using Equation (3.8) we can now also interpret the ratio of design fractions (3.7) as an importance-weighting factor relating LC costs fraction to time fraction.

We now determine where the LC costs extrema occur. Since from the restriction (2.4) all variation points must lie within an "acceptable" range and (from Equation (3.9)) the fractional LC costs increase varies linearly with variations ϵ_j, the maximum and minimum must occur on corners of the hyper-cuboid given by $\epsilon_j \in [\epsilon_j^-, \epsilon_j^+]$ for $j = 1, 2, \ldots, k$. To concisely write down expressions for the minimum and maximum of the LC costs relation given by Equation (3.9), we need to partition the set of all modified flight conditions. Let m (where $m \leq k$) be the number of flight conditions that have a negative amplification factor. Then partition the flight conditions so that the first m flight conditions are negative ($\gamma_j < 0$ for $j = 1, 2, \ldots, m$), while the last $k - m$ flight conditions (the remaining ones) are non-negative ($\gamma_j \geq 0$ for $j = m+1, m+2, \ldots, k$). Again there is no loss of generality in using this ordering, since we may order the flight conditions in any way we desire. Then the minimum and maximum are given respectively by

$$\min\left(\frac{\widehat{\$}}{\$} - 1\right) = \sum_{j=1}^{m} \gamma_j \epsilon_j^+ + \sum_{j=m+1}^{k} \gamma_j \epsilon_j^- \tag{3.10}$$

and

$$\max\left(\frac{\widehat{\$}}{\$} - 1\right) = \sum_{j=1}^{m} \gamma_j \epsilon_j^- + \sum_{j=m+1}^{k} \gamma_j \epsilon_j^+,$$

and occur respectively at the hyper-cuboid corners

$$< \epsilon_1^+, \epsilon_2^+, \ldots, \epsilon_m^+, \epsilon_{m+1}^-, \ldots, \epsilon_k^- > \qquad \text{(minimum)} \tag{3.11}$$

and

$$< \epsilon_1^-, \epsilon_2^-, \ldots, \epsilon_m^-, \epsilon_{m+1}^+, \ldots, \epsilon_k^+ > \qquad \text{(maximum).} \tag{3.12}$$

Note that the zero amplification factor case ($\gamma = 0$) was arbitrarily partitioned into the positive amplification factor set. We defer further explanation of this special case until the end of this section.

We mentioned earlier that condition (2.4), which if we divide by T may be re-written as

$$\sum_{j=1}^{k} \tau_j (1 + \epsilon_j^+) \leq 1, \tag{3.13}$$

could be relaxed. Figure 3.1 illustrates, for the case of two flight conditions being modified, the above restriction (shown as the contoured rectangle) along with the allowable region

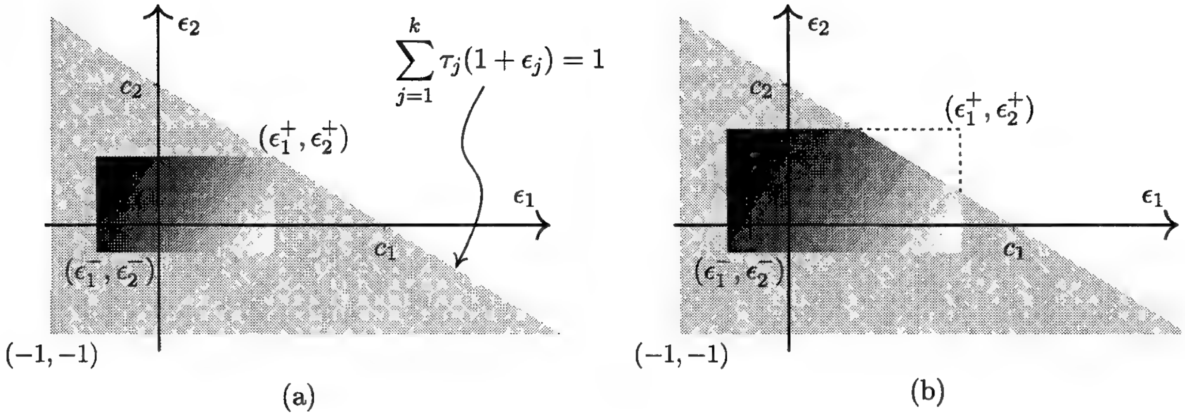

Figure 3.1: The bounds on the time fraction increases ϵ_1 and ϵ_2 (modifying two flight conditions). The light grey triangle represents allowable ranges, while the inner contoured rectangle illustrates the restriction given by Equation (3.13), where the shading variations in the rectangle represents LC costs (or fatigue damage) contours.

in which ϵ_1 and ϵ_2 may vary (light grey triangle). The contours inside the rectangle represent constant values of LC costs (or fatigue damage) and, as can be seen, these contours vary linearly with ϵ_1 and ϵ_2 as predicted by Equation (3.9). Figure 3.1(a) shows the restriction we have enforced using condition (3.13). We see from the contours that both the maximum and minimum occur at the corners of the rectangle. In Figure 3.1(b) we relax the restriction given by Equation (3.13), but now the region of ϵ_1-ϵ_2-variation is no longer rectangular. Since the triangle represents the allowable range for both ϵ_1 and ϵ_2 the rectangle must be truncated to fit within this triangular region. Under this relaxed condition the extrema can no longer occur at the rectangle corner $(\epsilon_1^+, \epsilon_2^+)$, but can instead occur at the intersection between the rectangle and triangle.

The constants c_1 and c_2 are easily obtained by changing the inequality in condition (3.13) to an equality. For the two-dimensional case (that is, varying two flight conditions) this results in

$$\tau_1(1+\epsilon_1) + \tau_2(1+\epsilon_2) = 1, \tag{3.14}$$

and hence

$$\epsilon_2 = (1 - \tau_1 - \tau_2 - \tau_1\epsilon_1)/\tau_2.$$

The above expression is the equation of the triangle's hypotenuse. Setting ϵ_1 to zero gives c_2 (and c_1 can be similarly derived to yield)

$$c_1 = (1 - \tau_1 - \tau_2)/\tau_1 \qquad \text{and} \qquad c_2 = (1 - \tau_1 - \tau_2)/\tau_2.$$

It is interesting to note that Equation (3.14), and its generalisation $\sum_{j=1}^{k} \tau_j(1+\epsilon_j) = 1$, can be obtained by setting the average variation in Equation (2.7) to $\epsilon = -1$. That is, the hypotenuse of the triangle represents the location on the ϵ_1-ϵ_2 plane where all other flight conditions have zero fractional time. Similarly, the vertical line of the triangle ($\epsilon_1 = -1$) represents the region in ϵ_1-ϵ_2 plane where the first flight condition has zero fractional

time. While the horizontal line of the triangle ($\epsilon_2 = -1$) represents the region in ϵ_1-ϵ_2 plane where the second flight condition has zero fractional time. These lower bounds on the fractional time variations were required by Equation (2.3), namely $\epsilon_j^- \geq -1$.

The contours of the rectangular region in Figure 3.1(a) show that the extrema occur on opposite sides of the rectangle. However, this statement is violated if the contours inside the rectangle are horizontal or vertical. Horizontal contour lines imply that the first flight condition has no effect (on the cost or damage, depending on what is being plotted), that is, $\gamma_1 = 0$. Similarly, vertical contour lines imply that the second flight condition has no effect (on the cost or damage), that is, $\gamma_2 = 0$. The direction of the contour lines is dependent upon the two amplification factors γ_1 and γ_2.

Given the above geometric interpretation of a zero amplification factor we can now pursue the issue of the special case $\gamma = 0$ on extrema locations. A more thorough, but also more complicated, partitioning of the amplification factors would be to divide them into three sets; negative, zero, and positive amplification factor sets. The extrema locations are then as given above in Equations (3.11) and (3.12). However, for any zero amplification factor ($\gamma_j = 0$) the locations of the minimum and maximum are independent of that particular flight condition. Hence, for $\gamma_j = 0$ any $\epsilon_j \in [\epsilon_j^-, \epsilon_j^+]$ would minimise or maximise the cost.

Finally, all the above statements on bounds generalise from the two-dimensional case to the n-dimensional case, where instead of a rectangle and triangle we respectively have a hyper-cuboid and hyper-tetrahedron.

4 Examples of Spectrum Modification

In the following example we will modify three flight conditions: level flight, turns, and ground-air-ground (gag) cycles. We first determine what effect fixed modifications have, and then we solve for the optimum change under bounded constraints for the same three flight conditions.

We can easily calculate α, the ratio of design fractions, for these three ($k = 3$) flight conditions. If we let $j = 1$, $j = 2$, and $j = 3$ refer to the flight conditions of level flight, turns, and gag cycles respectively, then from Equations (3.2) and (3.4) with values from Table 2.1 we have that

$$\begin{aligned}\sigma &= \sum_{j=1}^{k} \frac{\$_j}{\$} \\ &= \frac{1}{72.7}(1.75 + 13.9 + 29.0) \\ &= 0.614\end{aligned}$$

and

$$\begin{aligned}\tau &= \sum_{j=1}^{k} \frac{t_j}{T} \\ &= \frac{1}{100}(71.4 + 9.91 + 0.0309) \\ &= 0.813.\end{aligned}$$

Hence from Equation (3.7) the ratio of design fractions is

$$\alpha = \frac{1-\sigma}{1-\tau} = 2.06.$$

Using Equation (3.8) the amplification values, $\gamma_j = (\$_j/\$) - \alpha_k(t_j/T)$, are

$$\begin{aligned}\gamma_1 &= \frac{1.75}{72.7} - 2.06\frac{71.4}{100} \\ &= -1.45,\end{aligned}$$

$$\begin{aligned}\gamma_2 &= \frac{13.9}{72.7} - 2.06\frac{9.91}{100} \\ &= -0.0129,\end{aligned}$$

and

$$\begin{aligned}\gamma_3 &= \frac{29.0}{72.7} - 2.06\frac{0.0309}{100} \\ &= 0.398.\end{aligned}$$

As an example, if we increase level flight by 8%, increase turns by 15%, and decrease gag cycles by 12%, then from Equation (3.9) the new spectrum's LC costs will change by

$$\begin{aligned}
\frac{\widehat{\$}}{\$} - 1 &= \sum_{j=1}^{k} \gamma_j \epsilon_j \\
&= (-1.45) \times (0.08) + (-0.0129) \times (0.15) + (0.398) \times (-0.12) \\
&= -0.166.
\end{aligned}$$

In other words, LC costs due to fatigue damage experienced by components in the new usage spectrum will decrease by 16.6% when compared to the design spectrum. Again note that we have only used information from the modified flight conditions. The average variation, from Equation (2.7), is $\epsilon = -0.385$, so the amount of time spent in the remaining flight conditions decreases by 38.5%.

Let us now determine the minimum LC costs achievable using the ranges $\epsilon_1 = [-0.100, 0.150]$ (for level flight), $\epsilon_2 = [-0.050, 0.250]$ (for turns), and $\epsilon_3 = [-0.200, 0.070]$ (for gag cycles). From Equation (3.11) the configuration that would minimise LC costs is a 15% increase in level flight (since $\gamma_1 < 0$), a 25% increase in turns (since $\gamma_2 < 0$), and a 20% decrease in gag cycles (since $\gamma_3 \geq 0$). Under these conditions, the LC costs would decrease by 30.0%, that is, using Equation (3.10) we have that

$$\begin{aligned}
\min\left(\frac{\widehat{\$}}{\$} - 1\right) &= \sum_{j=1}^{m} \gamma_j \epsilon_j^{+} + \sum_{j=m+1}^{k} \gamma_j \epsilon_j^{-} \\
&= (-1.45) \times (0.150) + (-0.0129) \times (0.250) + (0.398) \times (-0.200) \\
&= -0.300.
\end{aligned}$$

As can be seen from the above examples, it is the values of the amplification factors that are important in calculating spectrum modification effects.

5 Special Cases

In this section we will consider three special cases of the general solution of § 3: modifying flight conditions by the same amount, modifying only one flight condition, and modifying all flight conditions except one.

If we modify any number of flight conditions by the same amount, say $\bar{\epsilon}$, then the new total LC costs fraction (3.3) and new total time fraction (3.5) simplify to

$$\widehat{\sigma} - \sigma = \sum_{j=1}^{k} \frac{\$_j}{\$}\epsilon_j = \bar{\epsilon}\sigma \qquad \text{and} \qquad \widehat{\tau} - \tau = \sum_{j=1}^{k} \frac{t_j}{T}\epsilon_j = \bar{\epsilon}\tau$$

respectively. Thus the new spectrum's LC costs change (3.6) may be re-written as

$$\begin{aligned} \frac{\widehat{\$}}{\$} - 1 &= (\widehat{\sigma} - \sigma) - \alpha_k(\widehat{\tau} - \tau) \\ &= \bar{\epsilon}(\sigma - \alpha_k\tau) \\ &= \bar{\epsilon}\left[\sigma - \left(\frac{1-\sigma}{1-\tau}\right)\tau\right] \\ &= \bar{\epsilon}\left(\frac{\sigma-\tau}{1-\tau}\right), \end{aligned} \tag{5.1}$$

where we have used Equation (3.6) to substitute for the design ratio α_k. Alternatively from Equation (3.9)

$$\begin{aligned} \frac{\widehat{\$}}{\$} - 1 &= \sum_{j=1}^{k} \gamma_j\epsilon_j \\ &= \bar{\epsilon}\gamma, \end{aligned} \tag{5.2}$$

where γ is the *total amplification factor* defined by

$$\begin{aligned} \gamma &= \sum_{j=1}^{k} \gamma_j \\ &= \sum_{j=1}^{k} \left(\frac{\$_j}{\$} - \alpha_k\frac{t_j}{T}\right) \end{aligned}$$

(we have used Equation (3.8) in going from the first to the second line in the above equations).

The sign of the total amplification factor determines whether the LC costs increase or decrease. Furthermore, from Equations (5.1) and (5.2) we note that the total amplification factor is

$$\gamma = \frac{\sigma-\tau}{1-\tau} \qquad (\text{if } \epsilon_j = \bar{\epsilon} \text{ for } j = 1, 2, \ldots, k).$$

A plot of the total amplification factor (that is the increase in LC costs as a fraction of $\bar{\epsilon}$) is shown in Figure 5.1. We see that for $\bar{\epsilon}$ positive the LC costs will increase whenever

the total LC costs fraction is greater than the total time fraction ($\sigma > \tau$). We also see that the largest LC cost changes occur when the total LC costs fraction is close to unity. For example, if the flight conditions we choose to modify have a LC costs fraction of 5% but a total time fraction of 80%, then a 10% increase in these flight conditions would reduce LC costs by 37.5%.

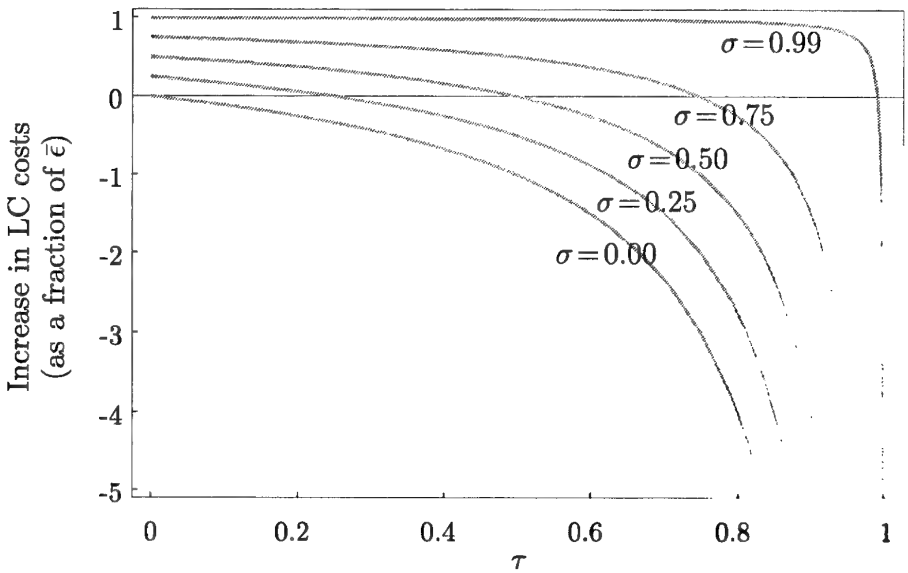

Figure 5.1: A plot of the total amplification factor, $(\sigma - \tau)/(1 - \tau)$, against total fractional time τ for several values of total fractional life-cycle costs, σ. For $\bar{\epsilon} > 0$ life-cycle costs will increase if $\sigma > \tau$.

If only *one* flight condition is modified, say the jth flight condition, then the total fractional LC costs and total fractional time simplify to

$$\sigma = \frac{\$_j}{\$} = \sigma_j \qquad \text{and} \qquad \tau = \frac{t_j}{T} = \tau_j \qquad (\text{for } k = 1),$$

respectively. Hence the LC costs variation becomes

$$\frac{\widehat{\$}}{\$} - 1 = \epsilon_j \left(\frac{\sigma_j - \tau_j}{1 - \tau_j} \right) \qquad (\text{for } k = 1). \tag{5.3}$$

The above expression is similar to Equation (5.1), which shows the result of modifying several flight conditions all with constant variation, and hence Figure 5.1 may be used to determine the effects of spectrum changes.

The other useful case to investigate is when $J - 1$ flight conditions (that is, all but one flight condition) are modified. In this way the ratio of design fractions α will be constant. Let us modify all flight condition except for the Jth flight condition (which is automatically adjusted by the average variation ϵ), then from the expression below Equation (3.7) we

have that the ratio of design fractions is

$$\alpha = \frac{\sigma_J}{\tau_J}. \tag{5.4}$$

Notice that to use this expression in a meaningful way, the flight condition we choose to be automatically adjusted must have a non-zero time fraction ($\tau_J \neq 0$). For example, using Table 2.1 we could not choose the "taxi" flight condition, otherwise the resulting ratio of design fractions would be infinite. Also notice that if the flight condition we choose to be automatically adjusted has a zero cost fraction ($\sigma_J = 0$), then the ratio of design fractions will be zero ($\alpha = 0$). If $\alpha = 0$, then the amplification factors of a flight condition will be equal to the cost fractions of that flight condition (that is, $\gamma_j = \sigma_j$, for $j = 1, 2, \ldots, J-1$ if $\sigma_J = 0$, refer to Equation (3.8)).

6 Fatigue Damage Analysis

As stated in the introduction, this analysis is not confined to LC costs. Since the LC costs equations are based on the fatigue damage incurred by each component (see Table 2.1) we can then show that the same procedure allows a comparison of the flight condition importance with respect to component fatigue damage. Due to the similarity in the procedure only the results are stated.

We first define the fatigue damage of the ith component under design and modified spectra respectively as

$$D_i = \sum_{j=1}^{J} D_{ij} \qquad \text{and} \qquad \widehat{D}_i = \sum_{j=1}^{J} D_{ij}(1+\epsilon_j).$$

We can now naturally define a *normalised fatigue damage* and hence *total normalised fatigue damage* of the ith component respectively as

$$\Delta_{ij} = \frac{D_{ij}}{D_i} \qquad \text{and} \qquad \Delta_i = \sum_{j=1}^{k} \Delta_{ij}.$$

Again note that the total normalised fatigue damage is a sum over modified flight conditions only. There are analogous definitions for the *new normalised fatigue damage* and the *new total normalised fatigue damage* denoted by $\widehat{\Delta}_{ij}$ and $\widehat{\Delta}_i$ respectively.

The increase in fatigue damage of the ith component may be written in one of the following two complementary forms (as shown earlier in Equations (3.6) and (3.9))

$$\frac{\widehat{D}_i}{D_i} - 1 = \left(\widehat{\Delta}_i - \Delta_i\right) - \omega_i\left(\widehat{\tau} - \tau\right) \tag{6.1}$$

or

$$\frac{\widehat{D}_i}{D_i} - 1 = \sum_{j=1}^{k} \kappa_{ij}\epsilon_j, \tag{6.2}$$

where ω_i and κ_{ij} are the ith component *weighting factor* and *amplification factor* (for the jth flight condition) respectively given by

$$\omega_i = \frac{1-\Delta_i}{1-\tau} \qquad \text{and} \qquad \kappa_{ij} = \Delta_{ij} - \omega_i\tau_j. \tag{6.3}$$

As before, the amplification factor is independent of time variations ϵ_j (but dependent on the set of flight conditions to be modified). Equations (6.1) and (6.2) have the same form as Equations (3.8) and (3.9) respectively. As such all comments made in § 3 hold true for the fatigue damage increase in components. This is especially true of the plots in Figure 5.1 where σ becomes Δ_i and instead of an increase in LC costs the vertical axis is a measure of the increase in fatigue damage of the ith component. Similarly Equation (5.3) becomes

$$\frac{\widehat{D}_i}{D_i} - 1 = \epsilon_j\frac{\Delta_j - \tau_j}{1-\tau_j} \qquad \text{(for } k=1\text{).} \tag{6.4}$$

Table 6.1: Amplification factors (sorted from maximum to minimum) for both component fatigue damage and life-cycle costs. Only one flight condition is modified at a time. The above values of fractional increase in life-cycle costs and fatigue damage were derived using Equations (5.3) *and* (6.4) *respectively.*

Fatigue damage amplification factors										**Life-cycle cost amplification factors**	
Main rotor hub		**Main supp. bridge**		**Main rotor blade**		**Lateral bell crank**		**Expand. blade pin**			
dsp	2.61E-1	turns	6.38E-1	gag cycle	9.78E-1	turns	3.54E-1	gag cycle	3.61E-1	gag cycle	3.99E-1
gag cycle	2.03E-1	ppd	6.07E-2	pullouts	1.11E-2	dive	1.08E-1	ppd	1.62E-1	dsp	1.16E-1
reversals	8.20E-2	pullouts	5.61E-2	heavy m.	7.84E-3	pullouts	9.74E-2	pullouts	1.37E-1	turns	1.02E-1
side/rear	8.01E-2	reversals	2.46E-2	taxi	0.00E+0	ppd	4.07E-2	dsp	1.12E-1	reversals	4.08E-2
sideslip	6.19E-2	gag cycle	2.11E-2	dsp	-1.39E-3	gag cycle	2.74E-2	turns	4.66E-2	pullouts	3.89E-2
pullouts	4.77E-2	dive	8.68E-3	landing	-3.63E-3	taxi	0.00E+0	autos	4.03E-2	side/rear	2.73E-2
hover	3.77E-2	taxi	0.00E+0	reversals	-5.07E-3	heavy m.	-1.40E-6	taxi	0.00E+0	sideslip	2.21E-2
ppd	2.77E-2	heavy m.	-1.40E-6	sideslip	-1.01E-2	dsp	-1.39E-3	heavy m.	-1.40E-6	ppd	1.93E-2
landing	2.05E-2	dsp	-1.39E-3	side/rear	-1.52E-2	landing	-3.63E-3	reversals	-2.24E-4	landing	7.15E-3
take off	1.06E-2	landing	-3.63E-3	take off	-1.80E-2	reversals	-5.07E-3	landing	-3.63E-3	hover	3.68E-3
heavy m.	2.28E-3	sideslip	-1.01E-2	autos	-1.81E-2	sideslip	-1.01E-2	sideslip	-1.01E-2	heavy m.	3.44E-3
taxi	0.00E+0	autos	-1.27E-2	hover	-2.38E-2	side/rear	-1.52E-2	side/rear	-1.52E-2	taxi	0.00E+0
autos	-1.81E-2	side/rear	-1.52E-2	dive	-2.38E-2	take off	-1.80E-2	take off	-1.80E-2	take off	-5.22E-3
dive	-2.38E-2	take off	-1.80E-2	ppd	-2.56E-2	autos	-1.81E-2	hover	-2.38E-2	dive	-1.53E-2
turns	-3.67E-2	hover	-2.38E-2	climb	-4.38E-2	hover	-2.38E-2	dive	-2.38E-2	autos	-1.67E-2
climb	-4.38E-2	climb	-4.38E-2	turns	-1.10E-1	climb	-4.38E-2	climb	-4.38E-2	climb	-4.38E-2
level flt	-2.49E+0	level flt	-2.16E+0	level flt	-2.49E+0	level flt	-1.58E+0	level flt	-2.49E+0	level flt	-2.41E+0

Table 6.1 provides a list of amplification factors (based on the fatigue damage values from Table 2.1), for both component fatigue damage and LC costs. Negative amplification factors are shown with a light grey background shading.

Level flight appears to be the most perturbation-sensitive flight condition. For example, a 10.0% increase in level flight (and associated 24.9% decrease in the remaining flight conditions) *decreases* main support bridge fatigue damage by 21.6%. As discussed earlier this high sensitivity is due to the large fraction of flight time that level flight consumes. Interestingly, the order of flight condition sensitivity is highly component-dependent. Consider turns as an example. A 10.0% increase in turns (and associated 1.10% decrease in the remaining flight conditions) decreases fatigue damage for the main rotor hub and blades, but *increases* fatigue damage for the main support bridge, lateral bell crank, and expandable blade pins. So we see that Table 6.1 not only provides a qualitative "importance" ordering of flight conditions in terms of fatigue damage of LC costs, but also provides a quantitative assessment of how "sensitive" flight conditions are to perturbations.

The only flight condition that amplifies perturbation effects (that is, $\gamma > 1$) is level flight. (Ground-air-ground cycles come close with $\gamma = 0.978$ for the fatigue damage to the main rotor blade.) All other flight conditions vary either LC costs or fatigue damage by a smaller fraction than the perturbation in time fraction of that flight condition (that is, $\gamma < 1$). For example, a 10% increase in turns increases main support bridge fatigue damage by 6.38%, which is below the time fraction perturbation of 10%. In fact, a 10% change in most flight conditions would lead to less than a 1% change in fatigue damage or LC costs.

The amplification factors listed in Table 6.1 cannot be simply added to determine successive perturbations. For example, we could *not* say that the total LC costs change of a 5% increase in level flight and a 10% decrease in turns is simply $-2.41 \times 0.05 + 0.0102 \times 0.10$. This problem is solvable (as we show below) but the solution procedure is far from trivial. However, this is an important aspect since changes in the time fraction of one flight condition may require changes in time fractions of other flight conditions. For example, changing the time fraction of the landings flight condition almost certainly must be accompanied by a change in the gag cycles flight condition.

Table 6.2 shows J flight conditions, and we wish to modify only flight conditions 1 and 2 by ϵ_1 and ϵ_2 respectively, the remaining flight conditions are automatically modified by an average amount ϵ_0. The column labelled "reqd" shows this required change. We attempt to do this by affecting two single changes (both unknown as yet). In "change A" we modify flight condition 1 by an unknown amount ϵ_A (and the remaining flight conditions are automatically adjusted by an unknown average variation ϵ_a). In "change B" we modify flight condition 2 by an unknown amount ϵ_B (and the remaining flight conditions are automatically adjusted by an unknown average variation ϵ_b). The required change is merely a sum of these two single changes.

Taking the two equations for ϵ_1 and ϵ_2 we have that

$$\epsilon_1 = \epsilon_A + \epsilon_b \qquad \text{and} \qquad \epsilon_2 = \epsilon_a + \epsilon_B. \tag{6.5}$$

Using Equation (2.7) we know that the average variations ϵ_a and ϵ_b are given respectively

Table 6.2: Simultaneously varying two flight conditions.

flight cond.	Changes A	B	reqd=A+B
1	ϵ_A	ϵ_b	$\epsilon_1 = \epsilon_A + \epsilon_b$
2	ϵ_a	ϵ_B	$\epsilon_2 = \epsilon_a + \epsilon_B$
3	ϵ_a	ϵ_b	$\epsilon_0 = \epsilon_a + \epsilon_b$
4	ϵ_a	ϵ_b	$\epsilon_0 = \epsilon_a + \epsilon_b$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
J	ϵ_a	ϵ_b	$\epsilon_0 = \epsilon_a + \epsilon_b$

by

$$\epsilon_a = -\frac{\tau_1 \epsilon_A}{1-\tau_1} \qquad \text{and} \qquad \epsilon_b = -\frac{\tau_2 \epsilon_B}{1-\tau_2}. \tag{6.6}$$

Substituting these expressions into Equation (6.5) and solving for ϵ_A and ϵ_B gives

$$\epsilon_A = \frac{(1-\tau_1)(1-\tau_2)}{(1-\tau_1-\tau_2)}\left[\epsilon_1 + \left(\frac{\tau_2}{1-\tau_2}\right)\epsilon_2\right] \tag{6.7}$$

and

$$\epsilon_B = \frac{(1-\tau_1)(1-\tau_2)}{(1-\tau_1-\tau_2)}\left[\epsilon_2 + \left(\frac{\tau_1}{1-\tau_1}\right)\epsilon_1\right]. \tag{6.8}$$

From Table 6.2 we also know that required average variation is given by $\epsilon_0 = \epsilon_a + \epsilon_b$. Using Equations (6.6) we have that

$$\epsilon_0 = -\frac{\tau_1 \epsilon_A}{1-\tau_1} - \frac{\tau_2 \epsilon_B}{1-\tau_2}.$$

Substituting Equations (6.7) and (6.8) into the above expression gives

$$\epsilon_0 = -\frac{\tau_1\epsilon_1 + \tau_2\epsilon_2}{1-\tau_1-\tau_2}.$$

Notice that the average variation ϵ_0 is exactly what we would have arrived at using Equation (2.7).

Using the above analysis (Table 6.2 and Equations (6.7) and (6.8)) we have shown that the amplification factors given in Table 6.1 are not additive in a straight forward manner.

We conclude this section by providing a table of amplification factors when all but one flight condition are modified. We choose *level flight* to be adjusted automatically by the average variation ϵ (that is, the only manually unmodified flight condition). Level flight is really a "default" flight condition in that any time not taken up by other flight conditions is considered level flight, and thus it makes sense to set level flight as the flight condition to be automatically adjusted. Table 6.3 lists the amplification factors when level flight is adjusted automatically by the average variation. Notice that the amplification factor for

Table 6.3: Amplification factors (sorted from maximum to minimum) for both life-cycle costs and component fatigue damage. The above values of fractional increase in life-cycle costs and fatigue damage were derived using Equations (3.8) and (6.3) respectively. The design ratio was obtained from Equation (5.4). Level flight is automatically adjusted (and accounted for) and hence not included in the above table.

Fatigue damage amplification factors										Life-cycle cost amplification factors	
Main rotor hub		Main supp. bridge		Main rotor blade		Lateral bell crank		Expand. blade pin			
dsp	2.62E-1	turns	6.61E-1	gag cycle	9.78E-1	turns	3.82E-1	gag cycle	3.61E-1	gag cycle	3.99E-1
gag cycle	2.03E-1	ppd	8.09E-2	pullouts	1.42E-2	dive	1.20E-1	ppd	1.83E-1	turns	1.88E-1
side/rear	9.39E-2	pullouts	5.85E-2	heavy man	7.84E-3	pullouts	9.91E-2	turns	1.41E-1	dsp	1.17E-1
reversals	8.66E-2	reversals	2.88E-2	taxi	0.00E+0	ppd	5.55E-2	pullouts	1.40E-1	reversals	4.55E-2
sideslip	7.13E-2	dive	2.86E-2	take off	0.00E+0	gag cycle	2.76E-2	dsp	1.13E-1	ppd	4.30E-2
turns	6.61E-2	gag cycle	2.14E-2	hover	0.00E+0	taxi	0.00E+0	autos	5.74E-2	pullouts	4.17E-2
hover	6.00E-2	autos	2.94E-3	climb	0.00E+0	heavy m.	-5.10E-7	reversals	4.82E-3	side/rear	4.14E-2
ppd	5.20E-2	taxi	0.00E+0	ppd	0.00E+0	dsp	-5.07E-4	climb	0.00E+0	sideslip	3.15E-2
pullouts	5.07E-2	heavy m.	-1.86E-7	dive	0.00E+0	landing	-1.32E-3	dive	0.00E+0	hover	2.60E-2
take off	2.80E-2	dsp	-1.85E-4	autos	0.00E+0	reversals	-1.84E-3	heavy m.	0.00E+0	take off	1.19E-2
landing	2.41E-2	landing	-4.81E-4	dsp	0.00E+0	sideslip	-3.65E-3	hover	0.00E+0	landing	1.06E-2
heavy m.	2.28E-3	sideslip	-1.33E-3	landing	0.00E+0	side/rear	-5.47E-3	landing	0.00E+0	dive	7.54E-3
taxi	0.00E+0	side/rear	-1.99E-3	reversals	0.00E+0	take off	-6.44E-3	side/rear	0.00E+0	heavy m.	3.45E-3
climb	0.00E+0	take off	-2.34E-3	side/rear	0.00E+0	autos	-6.49E-3	sideslip	0.00E+0	autos	7.70E-4
dive	0.00E+0	hover	-3.08E-3	sideslip	0.00E+0	hover	-8.46E-3	take off	0.00E+0	taxi	0.00E+0
autos	0.00E+0	climb	-5.57E-3	turns	0.00E+0	climb	-1.53E-2	taxi	0.00E+0	climb	-1.41E-3

level flight is not listed in Table 6.3; it has already been taken into consideration in all the listed amplification factors. As before, negative amplification factors have a light grey background shading.

Unlike the amplification factors listed in Table 6.1 (where only one flight condition is modified), in Table 6.3 any number of flight conditions may be modified simultaneously (this includes modifying all the tabled flight conditions simultaneously). Furthermore, the amplification factors listed are additive (when multiplied by the fractional time change). For example, increasing turns by 10% and increasing hover by 15% results, from Table 6.3, in an increase in LC costs of

$$0.10 \times (1.88{\times}10^{-1}) + 0.15 \times (2.60{\times}10^{-2}) = 2.27{\times}10^{-2},$$

that is a 2.27% increase in cost, and an increase in main support bridge fatigue damage of

$$0.10 \times (6.61{\times}10^{-1}) + 0.15 \times (-3.08{\times}10^{-3}) = 6.56{\times}10^{-2}$$

that is a 6.56% increase in fatigue damage. The amount of time spent in level flight has decreased by 1.53%, enough to compensate the increase in flight time from turns and hover, so that Equation (2.5) is satisfied.

It is interesting to note that the ordering of amplification factors for both the cost and damage differ between Tables 6.1 and 6.3. In fact, the ordering can be shown to be dependent on the set of flight conditions that is adjusted automatically by the average variation. This ordering statement seems strange at first, since it indicates that the "importance" (in terms of cost and damage) is dependent on the flight conditions we automatically adjust. On reflection, however, this statement makes sense, since we would expect the amplification factor ordering to be different if we only change, for example, turns and modify all other flight conditions automatically as compared to changing turns and only automatically adjusting level flight.

As was the case with the amplification factors listed in Table 6.1, most flight conditions have little effect on damage or LC costs. In summary the only flight conditions that had a greater than 1% change in damage or LC costs for a 10% change in time fraction were:

main rotor hub: dsp (2.62%) & gag cycles (2.03%);

main support bridge: turns (6.61%);

main rotor blade: gag cycles (9.78%);

lateral bell crank: turns (3.82%) & dives (1.20%);

expandable blade pin: gag cycles (3.61%), ppd (1.83%), turns (1.41%), pullouts (1.40%), & dsp (1.13%);

LC costs: gag cycle (3.99%), turns (1.88%), & dsp (1.17%).

The figures shown in brackets next to the flight condition are the percentage changes in damage or LC costs for a 10% increase in that flight condition (and corresponding decrease in level flight). All the figures shown above were obtained from Table 6.3. We thus see that only six flight conditions of the seventeen listed in the design spectrum (Table 2.1) would have any significant effect for a 10% change in a particular flight condition.

The above statement must be tempered, however, with the fact that the time fractions associated with some flight conditions are small (for example, only 0.03086% of the time is spent in gag cycles). This means that any increase to gag cycles, for example, would probably be significant (that is, $\epsilon \gg 1$), resulting in a significant fatigue damage or LC costs change.

7 Conclusions

Perturbations in the design usage spectrum result in linear variations in both life-cycle (LC) costs and fatigue damage, and hence the optimum solution must lie on the allowable perturbations boundary.

As expected the flight conditions whose perturbations had the most impact were those that occupied large fractions of the flying time. Unexpectedly, however, variations to flight conditions with small fractional LC costs or fatigue damage could significantly affect the overall LC costs or fatigue damage of a component respectively. This effects was amplified as the fractional time in the flight condition increased. Two tables of amplification factors were provided, allowing not only a qualitative analysis ("importance" ordering), but also a quantitative analysis (how sensitive different flight conditions were to change).

In the simplified Black Hawk example considered in this report, perturbations to most flight conditions produced percentage changes in LC costs and fatigue damage that were less than the percentage change to the time fraction of the flight conditions. For example, a 10% time fraction change produced less than a 1% change to both LC costs and fatigue damage for most flight conditions. However, large time changes (greater than 100%) have the potential to significantly modify both LC costs and fatigue damage.

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19. ABSTRACT

The Australian Defence Force (ADF) operates rotary and fixed-wing aircraft in a different manner from the way that the aircraft manufacturers envisaged. That is, the usage spectra of the ADF aircraft are different from the usage spectra assumed by the aircraft manufacturer during design. The effects of perturbing the amount of time spent in flight conditions on both component fatigue damage and life-cycle costs are investigated in this report. An "amplification factor" is developed, which allows the effect of varying the amount of time spent in different flight conditions on both damage and cost to be determined. These amplification factors give both qualitative ("importance" ordering) and quantitative (sensitivity) information regarding all flight conditions. The resulting procedure is both easy to implement and use, and allows the operator to determine the effects of different spectra on both damage and cost. The outlined procedures will lead to cost savings and safety improvement for the ADF for both rotary and fixed-wing aircraft.

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